CBSE Board
Class XI Mathematics
Sample Paper – 4

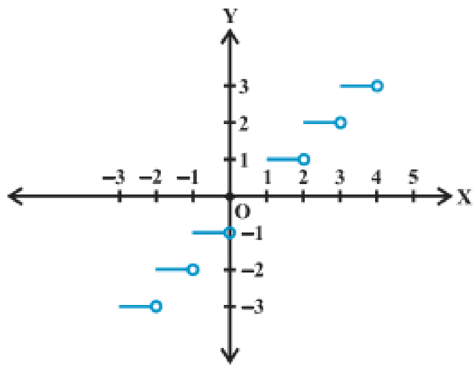
Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions.
3. Questions 1 – 4 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 5 – 12 in Section B are short-answer type questions carrying 2 mark each.
5. Questions 13 – 23 in Section C are long-answer I type questions carrying 4 mark each.
6. Questions 24 – 29 in Section D are long-answer type II questions carrying 6 mark each.

SECTION – A

1. Identify the function which the given graph represents.



2. If $4x + i(3x - y) = 3 + i(-6)$, where x and y are real numbers, then find x and y.

3. Let A = {1, 2} and B = {3, 4}. Find the number of relations from A to B.

OR

Find x and y if $(x + 3, 5) = (6, 2x + y)$

4. Write the negation of statement : "Australia is a continent."

SECTION – B

5. If $f : R \to R$ is defined by $f(x) = x/(x^2-1)$, find $f(f(2))$.

OR

If $f(x) = 3x^4 - 5x^2 + 9$ find $f(x - 1)$.

6. Solve : $\cos 3\theta + 8\cos^3\theta = 0$

7. Show that the roots of equation $(a^2+b^2)x^2-2b(a+c)x+(b^2+c^2)=0$ are real and equal if a,b,c are in GP.

8. Draw the graph of the function $|x + 2| - 1$.

9. Find the co-ordinates of the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$

OR

If the latus rectum of an ellipse is equal to half of minor axis, find its eccentricity.

10. In a survey of 600 students in a school, 150 students drink tea and 225 drink coffee, and 100 drink both tea and coffee. Find how many students drink neither tea nor coffee?

OR

In a group of 800 people, 550 can speak Hindi and 450 can speak English. How many can speak both Hindi and English?

11. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by $\{(a, b): a, b \in A, \text{a divides b}\}$

(i) Write in the roster form

(ii) Find the domain of R

(iii) Find the range of R

12. $f(x) = -1$ if $x < 0$

$= 1$ if $x > 0$

Draw the graph of the above function

SECTION – C

13. Represent the complex number $z = 1 + i\sqrt{3}$ in the polar form.

OR

Solve the following $\left[i^{18}+\frac{1}{i^{25}}\right]^2$

14. Find n given that, ${}^{n-1}P_3 : {}^{n}P_4 = 1 : 9$

15. Solve the given equation $2\cos^2 x + 3\sin x = 0$

16. Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

17. The income of a person is Rs. 3, 00, 000, in the first year and he receives an increment of Rs. 10, 000 to his income per year for the next 19 years. Find the total amount, he received in 20 years.

18. Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.

OR

Find the sum of the n terms of the series 5 + 11 + 19 + 29 + ...

19. Find the equation of a circle which passes through the points (2, −2), and (3, 4) and whose centre lies on the line x + y = 2.

20. Solve $\sqrt{5}x^2 + x + \sqrt{5} = 0$.

21. Find the r^{th} term from the end in the expansion of $(x + a)^n$.

OR

Find the coefficient of x^6y^3 in the expansion of $(x + 2y)^9$.

22. Prove by induction that the sum, $S = n^3 + 3n^2 + 5n + 3$, is divisible by 3 for all $n \in N$.

23. Prove that $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$

SECTION – D

24. Find the solution region for the following system of inequations:

$x + 2y \leq 10, x + y \geq 1, x - y \leq 0, x \geq 0, y \geq 0$

25. Find the mean deviation about the mean for the following continuous frequency distribution, using the short cut method for finding mean.

Marks Obtained	Number of Students
0 – 10	12
10 – 20	18
20 – 30	27
30 – 40	20
40 – 50	17
50 – 60	6

OR

The scores of 48 children in an intelligence test are shown in the following frequency table.

Calculate the variance σ^2 and find out the percentage of children whose scores lie between $\bar{x}-\sigma$ and $\bar{x}+\sigma$

Score	Frequency
71	4
76	3
79	4
83	5
86	6
89	5
92	4
97	4
101	3
103	3
107	3
110	2
114	2

26. Find the equations of the lines through the point (3, 2) which are at an angle of 45° with the line x – 2y = 3.

OR

The mid points of the sides of a triangle are (2, 1), (-5, 7) and (-5, -5). Find the equations of the sides of the triangle.

27. Find the derivative using the first principle of f(x), where f(x) is given by $f(x)=x+\frac{1}{x}$

28. For all n ≥ 1, prove using Principle of Mathematical Induction

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

29. One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be

(i) A diamond

(ii) An ace

(iii) A black card

(iv) Not a diamond

(v) Not a black card

(vi) Not an ace

OR

A committee of two persons is to be selected from two men and two women.
What is the probability that the committee will have (a) no man (b) one man (c) two men?

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Sample Paper – 4 Solution

SECTION – A

1. Such a function is called the greatest integer function.

From the definition $[x] = -1$ for $-1 \le x < 0$

$[x] = 0$ for $0 \le x < 1$

$[x] = 1$ for $1 \le x < 2$

$[x] = 2$ for $2 \le x < 3$ and so on.

2. $4x + i(3x - y) = 3 + i(-6)$

Equating real and the imaginary parts of the given equation, we get,

$4x = 3,\ 3x - y = -6,$

Solving simultaneously, we get $x = \frac{3}{4}$ and $y = \frac{33}{4}$

3. Given A = {1, 2} and B = {3, 4} A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}.

Since n(A × B) = 4, the number of subsets of A × B is 2^4.

Therefore, the number of relations from A to B will be 16.

OR

$(x + 3, 5) = (6, 2x + y)$

$x + 3 = 6$

$x = 3$ ....(i)

Also,

$2x + y = 5$

$6 + y = 5$ from (i)

$y = -1$

4. The negation of the given statement :

It is false that Australia is a continent.

Or

Australia is not a continent.

SECTION – B

5. Given

$$f(x)=\frac{x}{x^2+1}$$

$$\Rightarrow f(2)=\frac{2}{2^2+1}=\frac{2}{5}$$

$$\Rightarrow f(f(2))=f\left(\frac{2}{5}\right)=\frac{\frac{2}{5}}{\left(\frac{2}{5}\right)^2+1}=\frac{10}{29}$$

OR

$f(x) = 3x^4 - 5x^2 + 9$

$f(x-1) = 3(x-1)^4 - 5(x-1)^2 + 9$

$f(x-1) = 3(x^4 - 4x^3 + 6x^2 - 4x + 1) - 5(x^2 - 2x + 1) + 9$

$f(x-1) = 3x^4 - 12x^3 + 18x^2 - 12x + 3 - 5x^2 + 10x - 5 + 9$

$f(x-1) = 3x^4 - 12x^3 + 13x^2 - 2x + 7$

6.

given that $\cos 3\theta + 8\cos^3\theta = 0$

$\rightarrow 4\cos^3\theta - 3\cos\theta + 8\cos^3\theta = 0$

$\rightarrow 12\cos^3\theta - 3\cos\theta = 0$

$\rightarrow 3\cos\theta(4\cos^2\theta - 1) = 0$

$\rightarrow \cos\theta = 0 \quad \text{or} \quad (4\cos^2\theta - 1) = 0$

$\rightarrow \cos\theta = 0 \ldots\ldots\ldots\ldots \theta = (2n+1)\frac{\pi}{2}, n \in I$

$\rightarrow (4\cos^2\theta - 1) = 0 \ldots\ldots\ldots\ldots \cos\theta = \frac{1}{2} \ldots\ldots \theta = n\pi \pm \frac{\pi}{3}, n \in I$

7. Discriminant of given equation is 0…….real and equal

$0 = (-2b(a+c))^2 - 4(a^2+b^2)(b^2+c^2)$

$0 = 4(-b^4 - a^2c^2 + 2b^2ac)$

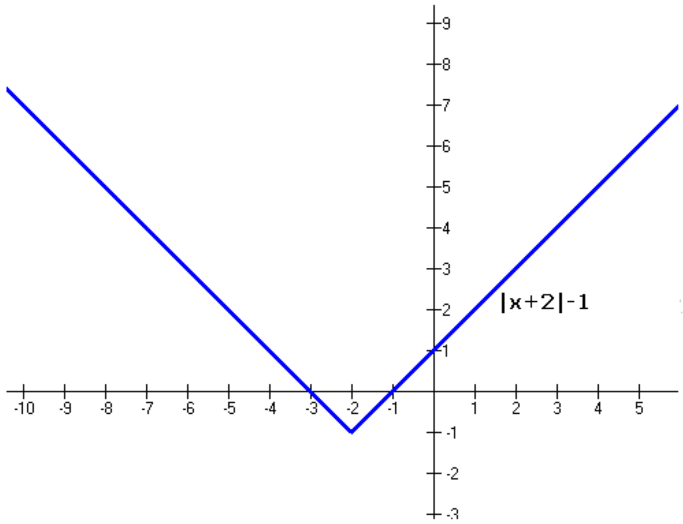
$0 = -4(b^2 - ac)^2$

$b = \sqrt{ac}$

hence they are in GP

8. Graph of function |x + 2| − 1

x	0	−2	−1	1	2
f(x)	1	−1	0	2	3



9. Since the denominator of x^2 is greater than the denominator of y^2, the major axis is along the x-axis. Comparing the given equation with the standard equation of the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

a = 5, b =3

$$ae=\sqrt{a^2-b^2}=\sqrt{25-9}=4$$

So the foci are (4, 0) and (−4, 0)

OR

Let the equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and e be the eccentricity.

Given that

Latus rectum = (minor axis)/2

$$\frac{2b^2}{a}=\frac{1}{2}\times 2b$$

$$2b=a$$

$$4b^2=a^2$$

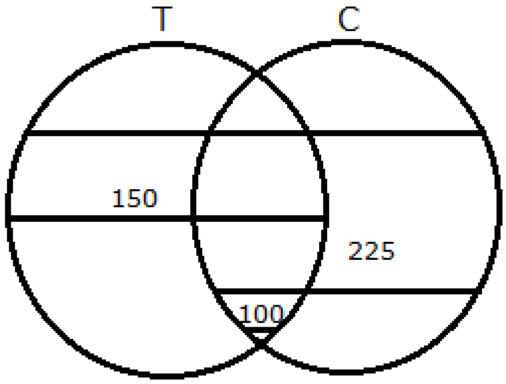
$$4a^2\left(1-e^2\right)=a^2$$

$$4-4e^2=1$$

$$e^2=\frac{3}{4}\Rightarrow e=\frac{\sqrt{3}}{2}$$

10. Let T represent the students who drink tea and C represents students who drink coffee and x represent the number of students who drink tea or coffee.

So, $x = n(T \cup C)$



$n(T) = 150; n(C) = 225$

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

Substituting the values,

$x = 150 + 225 - 100 = 275$

Number of students who drink neither tea nor coffee

$= 600 - 275 = 325.$

OR

Let H denote the set of people speaking Hindi and E denote the set of people speaking English.

$n(H) = 550, n(E) = 450$ and $n(H \cup E) = 800$

$n(H \cap E) = n(H) + n(E) - n(H \cup E)$

$= 550 + 450 - 800$

$= 200$

Hence, 200 persons can speak both Hindi and English.

11. R: { (a, b): a, b $\in$ A, a divides b}, Also

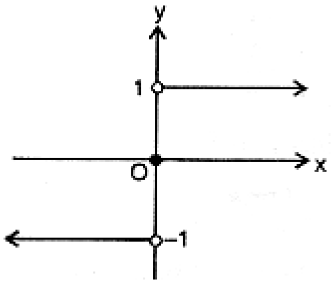
A = {l, 2, 3, 4, 6}

(i) R = {(1, 1), (1, 2), (1, 3), (1, 4),(1, 6), (2, 2),(2, 4), (2,6), (3, 3), (3, 6),(4, 4), (6, 6)}

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

12. Given Signum function:



The two branches
Bold bullet at (0, 0)
Circle at (0, 1) and (0, −1)

SECTION – C

13. Let $1 = r\cos\theta$, $\sqrt{3} = r\sin\theta$

By squaring and adding, we get,

$r^2(\cos^2\theta + \sin^2\theta) = 4, \Rightarrow r = 2$

$\cos\theta = \frac{1}{2}, \sin\theta = \frac{\sqrt{3}}{2}$, So $\theta = \frac{\pi}{3}$

Therefore, required polar form is

$z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

OR

$$\left[i^{18} + \frac{1}{i^{25}}\right]^2$$

$$= \left[i^{4\times4+2} + \frac{1}{i^{4\times6+1}}\right]^2 = \left[i^2 + \frac{1}{i}\right]^2$$

$$= \left(-1 + \frac{1}{i}\right)^2$$

$$= (-1)^2 + \left(\frac{1}{i}\right)^2 + 2(-1)\left(\frac{1}{i}\right)$$

$$= 1 + \frac{1}{i^2} - \frac{2}{i} = 1 + \frac{1}{-1} - \frac{2}{i}$$

$$= \frac{-2}{i} = \frac{2i^2}{i} = 2i = 0 + 2i$$

14.

$$^{n-1}P_3 : {}^{n}P_4 = 1:9$$

$$\frac{(n-1)!}{(n-4)!} : \frac{n!}{(n-4)!} = \frac{1}{9}$$

$$\Rightarrow \frac{(n-1)!}{(n-4)!} \times \frac{(n-4)!}{n!} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{9}$$

$$\Rightarrow n = 9$$

15. $2\cos^2 x + 3\sin x = 0$

Using $\cos^2 x = 1 - \sin^2 x$

$\Rightarrow 2\sin^2 x - 3\sin x - 2 = 0$

$\Rightarrow (\sin x - 2)(2\sin x + 1) = 0$

$\Rightarrow \sin x = 2$ (not possible) and $\sin x = -\frac{1}{2}$

$$\Rightarrow \sin x = -\sin\frac{\pi}{6} = \sin(\pi + \frac{\pi}{6})$$

$$\Rightarrow \sin x = \sin\frac{7\pi}{6}$$

$$x = n\pi + (-1)^n \frac{7\pi}{6}$$

16.

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$$

$$= \cot 3x$$

17. The sequence is an A.P. with a = 3, 00,000, d = 10,000, and n = 20.

Using the sum formula, we get,

S_{20} = (20/2) [600000 +19 × 10000]

= 10 (790000) = 79, 00,000.

Hence, the person received Rs. 79, 00,000 as the total amount at the end of 20 years.

18. Let G1, G2, and G3 be three numbers between 1 and 256 such that 1, G1, G2, G3, 256 is a G.P.

Now 1 is the Ist term and 256 is the 5th term of the G.P.

Let r be the common ratio of the G.P.

$256 = r^4$ giving $r = \pm 4$ (Taking real roots only)

For r = 4, we have $G1 = ar = 4$, $G2 = ar^2 = 16$, $G3 = ar^3 = 64$

Similarly, for r = –4, numbers are – 4, 16 and – 64.

As -4 and -64 doesn't fall between 1 and 256 hence $r \neq -4$.

Hence, we can insert, 4, 16, 64 between 1 and 256 so that the resulting sequences is in G.P.

OR

Applying the Method of Difference

$S_n = 5 + 11 + 19 + 29 + ... + a_{n-1} + a_n$

$S_n = 5 + 11 + 19 + ... + a_{n-2} + a_{n-1} + a_n$

On subtraction, we get,

$0 = 5 + [6 + 8 + 10 + 12 + ...(n - 1) \text{ terms}] - a_n$

$$a_n = 5 + \frac{(n-1)[12+(n-2)\times 2]}{2}$$

$$= 5 + (n - 1)(n + 4) = n^2 + 3n + 1$$

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} (k^2 + 3k + 1) = \sum_{k=1}^{n} k^2 + 3\sum_{1}^{n} k + n$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n$$

$$= \frac{n(n+1)}{2}\left(\frac{(2n+1)}{3} + 3\right) + n$$

$$= \frac{n(n+1)(2n+10)}{6} + n$$

19. Let the equation of the circle be $(x - h)^2 + (y - k)^2 = r^2$

Since the circle passes through (2, –2) and (3, 4), we have,

$(2 - h)^2 + (-2 - k)^2 = r^2$... (1)

and $(3 - h)^2 + (4 - k)^2 = r^2$... (2)

Also since the centre lies on the line x + y = 2, we have,

h + k = 2 ... (3).

Solving the equations (1), (2) and (3), we get,

h =0 .7, k = 1.3 and $r^2 = 12.58$

Hence, the equation of the required circle is

$(x - 0.7)^2 + (y - 1.3)^2 = 12.58$

20. Given equation is, $\sqrt{5}x^2 + x + \sqrt{5} = 0$..

$a = \sqrt{5}$, $b = 1$, $c = \sqrt{5}$

Discriminant of the equation is

$b^2 - 4ac = 1^2 - 4 \times \sqrt{5} \times \sqrt{5} = 1 - 20 = -19$

Therefore, the roots are

$$\frac{-1 \pm \sqrt{-19}}{2\sqrt{5}} = \frac{-1+\sqrt{19}\,i}{2\sqrt{5}}, \frac{-1-\sqrt{19}\,i}{2\sqrt{5}}$$

21. There are (n + 1) terms in the expansion of $(x + a)^n$

Observing the terms we can say that the first term from the end is the last term, i.e., (n + 1)th term of the expansion and n + 1 = (n + 1) – (1 – 1).

The second term from the end is the nth term of the expansion, and n = (n + 1) – (2 – 1).

The third term from the end is the (n – 1)th term of the expansion and n – 1 = (n + 1)– (3 – 1) and so on.

Thus rth term from the end will be term number (n + 1) – (r – 1) = (n – r + 2)th of the expansion.

the (n – r + 2)th term is $^nC_{n-r+1} \times x^{r-1} \times a^{n-r+1}$

OR

$T_{r+1} = {}^9C_r\, x^{9-r} (2y)^r = {}^9C_r 2^r \,.\, x^{9-r} \,.\, y^r$

Comparing the indices of x as well as y in x^6y^3 and in T_{r+1}, we get r = 3.

Thus, the coefficient of $x^6y^3$is

$$^9C_3 2^3 = \frac{9!}{3!6!}.2^3 = \frac{9\times8\times7}{3\times2}\times2^3 = 672$$

22.. Let P(n) be the statement given by

P(n): $Sn = n^3 + 3n^2 + 5n + 3$

Step I: When n = 1, P(1) = 13 + 3 × 12 + 5 × 1 + 3

Since, P(1) = 13 + 3 × 12 + 5 × 1 + 3 = 12, hence, it is divisible by 3.

∴ P(1) is true.

Step II: Let P(m) be true.

$Sm = m^3 + 3m^2 + 5m + 3$ is divisible by 3.

$Sm = m^3 + 3m^2 + 5m + 3 = 3\lambda$ for some $\lambda \in N$. ....(i)

We now wish to show that P(m + 1) is true. For this reason we have to show that

$(m + 1)^3 + 3(m + 1)^2 + 5(m + 1) + 3$ is divisible by 3.

$= (m^3 + 3m^2 + 3m + 1) + 3m2 + 9m + 9$

$= 3\lambda + 3(m^2 + 3m + 3)$

= 3(λ + m^2 + 3m + 3) ∈ N

= 3μ, where μ = λ + m2 + 3m + 3 ∈ N............[using (i)]

∴ P(m + 1) is true.

Thus P(m) is true ⇒ P(m + 1) is true.

Hence, by principle of mathematical induction the statement is true for all n ∈ N.

23. Consider L.H.S.

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= \cos\left(\frac{9\pi}{13} + \frac{\pi}{13}\right) + \cos\left(\frac{9\pi}{13} - \frac{\pi}{3}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= \cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= \cos\left(\pi - \frac{3\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= -\cos\frac{3\pi}{13} - \cos\frac{5\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$$

= R.H.S.

Thus L.H.S. = R.H.S.

SECTION – D

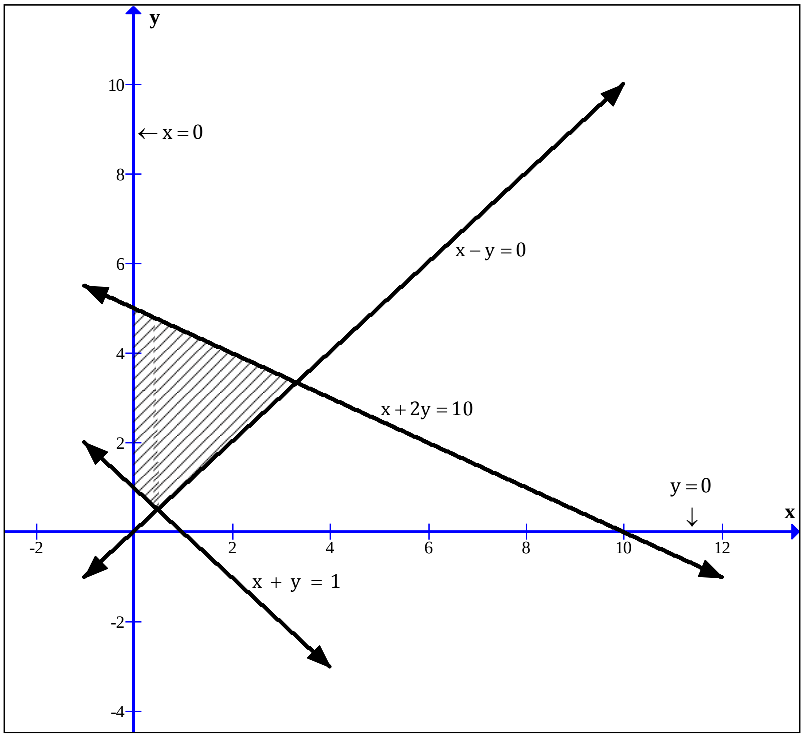
24. Given inequalities:

$x + 2y \leq 10$, $x + y \geq 1$, $x - y \leq 0$, $x \geq 0$, $y \geq 0$,

Consider the corresponding equations x + 2y = 10, x + y = 1 and x – y = 0.

On plotting these equations on the graph, we get the graph as shown.

Also we find the shaded portion by substituting (0, 0) in the inequations.



25.

Marks Obtained	Number of Students
0 – 10	12
10 – 20	18
20 – 30	27
30 – 40	20
40 – 50	17
50 – 60	6

Let assumed mean = a = 25

xi	fi	di = $\frac{x_i - a}{h}$	fi di
5	12	−2	−24
15	18	−1	−18
25	27	0	0
35	20	1	20
45	17	2	34
55	6	3	18
Total	100		30

$$\bar{x} = a + \frac{\sum_{i=1}^{n} f_i d_i}{N} \times h$$
$$= 25 + \frac{30 \times 10}{100}$$
$$= 28$$

OR

Let assumed mean = a = 90

xi	fi	di = xi - a	di2	fi di	fi di2
71	4	−19	361	−76	1444
76	3	−14	196	−42	588
79	4	−11	121	−44	484
83	5	−7	49	−35	245
86	6	−4	16	−24	96
89	5	−1	1	−5	5
92	4	2	4	8	16
97	4	7	49	28	196
101	3	11	121	33	363
103	3	13	169	39	507
107	3	17	289	51	867
110	2	20	400	40	800
114	2	24	576	48	1152
Total	48			21	6763

Mean:

$$\bar{x} = a + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i} \text{..........} \therefore \ \bar{x} = 90 + \frac{21}{48} = 90 + 0.44 = 90.44$$

Variance:

$$\sigma^2 = \frac{\sum_{i=1}^{n} f_i {d_i}^2}{\sum_{i=1}^{n} f_i} - \left(\frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i} \right)^2$$

$$\sigma^2 = \frac{6763}{48} - \left(\frac{21}{48}\right)^2$$

$$= 140.896 - 0.191 = 140.705(\text{nearly})$$

$$\text{Hence}, \sigma = \sqrt{140.705} = 11.86$$

$$\Rightarrow \bar{x} - \sigma = 90.44 - 11.86 = 78.58$$

$$\text{and } \bar{x} + \sigma = 90.44 + 11.86 = 102.30$$

The number of observations which lie between $\bar{x} - \sigma$ and $\bar{x} + \sigma$

= 4 + 5 + 6 + 5 + 4 + 4 + 3 = 31

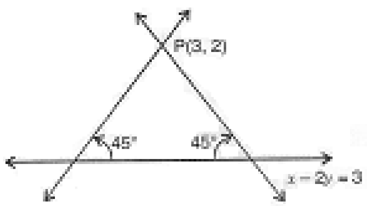
Percentage of these observations $= \frac{31}{48} \times 100 = 64.58$ (nearly)

26. Let the line through (3, 2) be y − 2 = m(x − 3) ... (i)

Slope of line x − 2y = 3 is $\frac{1}{2}$.

Now,

$$\tan(\pm 45^\circ) = \frac{m - \frac{1}{2}}{1 + \frac{m}{2}} \Rightarrow \pm 1 = \frac{2m - 1}{2 + m}$$



Case I:

$$\frac{2m - 1}{2 + m} = 1 \Rightarrow 2m - 1 = 2 + m$$

So, m = 3

Equation of line is y − 2 = 3(x − 3).

Therefore 3x − y − 7 = 0 is the required equation

Case II:

$$\frac{2m - 1}{2 + m} = -1 \Rightarrow 2m - 1 = -2 - m$$

3m = − 1

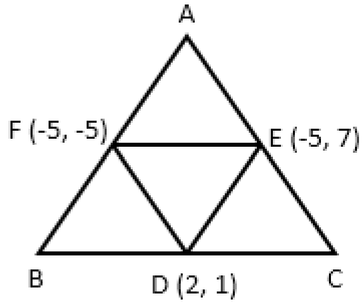
$$m = -\frac{1}{3}$$

Now the equation is

$$y - 2 = -\frac{1}{3}(x - 3)$$

3y − 6 = −x + 3 $\quad \therefore$ x + 3y − 9 = 0

OR



Let D(2, 1), E(-5, 7) and F(-5, -5) be the midpoints of sides BC, CA, and AB respectively. Of triangle ABC.

We know that the line joining the midpoints of two sides of a triangle is parallel to the third side.

DE is parallel to AB, EF parallel to BC, DE parallel to AC

Slope of AB = Slope of DE

Slope of BC = slope of EF and slope of AC = slope of DF

Let, m_1, m_2 and m_3 be the slopes of AB, BC and CA respectively.

m_1 = slope of AB = slope of DE = $\frac{7-1}{-5-2} = \frac{-6}{7}$

m_2 = slope of BC = slope of EF = $\frac{7+5}{-5+5}$ undefined

m_3 = slope of CA = slope of DF = $\frac{1+5}{2+5} = \frac{6}{7}$

Side AB passes through F(-5, -5) and has slope $m_1 = \frac{-6}{7}$

Its equation is

$y + 5 = \frac{-6}{7}(x + 5)$

$6x + 7y + 65 = 0$

Side BC is parallel to Y – axis and passes through D(2, 1).

Its equation is

$x = k$.

As it passes through (2, 1). $k = 2$.

Hence, equation BC is $x = 2$.

Side CA passes through (-5, 7) and has slope $m_3 = \frac{6}{7}$

Its equation is

$y - 7 = \frac{-6}{7}(x + 5)$

$6x - 7y + 79 = 0$

27. The function is not defined at x = 0.

At all other points we have,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left(x+h+\frac{1}{x+h}\right)-\left(x+\frac{1}{x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[h+\frac{1}{x+h}-\frac{1}{x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[h+\frac{x-x-h}{x(x+h)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[h\left(1-\frac{1}{x(x+h)}\right)\right]$$

$$= \lim_{h\to 0}\left[1-\frac{1}{x(x+h)}\right]$$

$$= 1-\frac{1}{x^2}$$

28. Let

$$P(n): \frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{n(n+1)} = \frac{n}{n+1}$$

$$P(1): \frac{1}{1.2} = \frac{1}{1+1},$$

which is true. Thus, P(n) is true for n = 1

Assume that P(k) is true for some natural number k,

$$P(k): \frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{k(k+1)} = \frac{k}{k+1}$$

We need to prove that P(k + 1) is true whenever P(k) is true.

We have, P(k + 1)=

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)} = \frac{(k+1)}{(k+2)}$$

R.H.S. of P(k) =

$$\left[\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{k(k+1)}\right]+\frac{1}{(k+1)(k+2)}$$

$$= \frac{k}{(k+1)}+\frac{1}{(k+1)(k+2)}$$

$$= \frac{k^2+2k+1}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2}$$

= R.H.S.

So P(k + 1) is true whenever P(k) is true. So the result holds for all natural numbers.

29. When a card is drawn from a well shuffled deck of 52 cards, the number of possible outcomes is 52.

(i) Let A be the event that 'the card drawn is a diamond'

Clearly the number of elements in set A is 13.

Therefore,

$$P(A)=\frac{13}{52}=\frac{1}{4}$$

i.e. Probability of drawing a diamond card $=\frac{1}{4}$

(ii) We assume that the event, 'card drawn is an ace' is B

Clearly the number of elements in set B is 4.

So,

$$P(B)=\frac{4}{52}=\frac{1}{13}$$

(iii) Let C denote the event that the 'card drawn is a black card'.

Therefore, number of elements in the set C = 26

$$P(C)=\frac{26}{52}=\frac{1}{2}$$

Thus, Probability of drawing a black card $=\frac{1}{2}$

(iv) We assumed in (i) above that A is the event when 'card drawn is a diamond', so the event 'card drawn is not a diamond' may be denoted as A' or 'not A' Now,

$$P(\text{not } A)=1-P(A)=1-\frac{1}{4}=\frac{3}{4}$$

(v) The event, 'card drawn is not a black card' may be denoted as C′ or 'not C'. We know that

$$P(\text{not } C)=1-P(C)=1-\frac{1}{2}=\frac{1}{2}$$

Therefore, Probability of not drawing a black card $=\frac{1}{2}$

(vi) The event that the 'card drawn is not an ace' may be denoted as B' or 'not B'. We know that

$$P(\text{not } B)=1-P(B)=1-\frac{1}{13}=\frac{12}{13}$$

Therefore the probability that a card drawn is not an ace $=\frac{12}{13}$

OR

The total number of persons = 2 + 2 = 4. Out of these four persons, two can be selected in $^4C_2$ ways.

a) No men in the committee of two means there will be two women in the committee. Out of two women, two can be selected in $^2C_2 = 1$ way

$$P(\text{no man}) = \frac{^2C_2}{^4C_2} = \frac{2}{4\times 3} = \frac{1}{6}$$

(b) One man in the committee means that there is one woman. One man out of 2 can be selected in $^2C_1$ ways and one woman out of 2 can be selected in $^2C_1$ ways.

Together they can be selected in $^2C_1 \times {}^2C_1$ ways.

$$P(\text{one man}) = \frac{^2C_1 \times {}^2C_1}{^4C_2} = \frac{2\times 2}{2\times 3} = \frac{2}{3}$$

(c) Two men can be selected in $^2C_2$ ways

$$P(\text{Two men}) = \frac{^2C_2}{^4C_2} = \frac{1}{^4C_2} = \frac{1}{6}$$